hep-ph/9503444
TTP95-15[1]
March 1995

TWO-LOOP LIGHT QUARK CORRECTIONS TO THE TOP WIDTH[2]

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Abstract

A method of computing two-loop fermionic contributions to the width of a heavy quark is described. An analytical formula for this effect in the limit of mass of the quark much larger than the decay products is obtained for the first time. The result confirms previous numerical studies.

1 Introduction

The recently discovered top quark attracts attention of many physicists who regard its extraordinarily large mass as a hint of its connection with "new physics". One consequence of the unusually big mass is the large width of the top quark Γ_t. This quantity is certainly sensitive to possible exotic particles with which top can interact and will be subject of future precision experimental studies. It is important to know the standard model prediction for Γ_t as precisely as possible and a lot of effort has been invested in studies of quantum effects which modify it. In the framework of the standard model the transition of the top quark into a bottom quark and a W boson is by far the dominant decay mode and it has been the focus of the recent studies. In particular,

[1]The complete postscript file of this preprint, including figures, is available via anonymous ftp at ttpux2.physik.uni-karlsruhe.de (129.13.102.139) as /ttp95-15/ttp95-15.ps or via www at http://ttpux2.physik.uni-karlsruhe.de/cgi-bin/preprints/ Report-no: TTP95-15

[2]Dedicated to Professor K. Zalewski to honor his 60th Birthday.

one-loop QCD corrections have been found to reduce $\Gamma(t \to bW)$ by about 10% [1]. Recently the first step has been made towards numerical calculation of two-loop strong corrections to this reaction, namely the subset of diagrams containing a fermion loop or an additional fermion pair in the final state [2]. The relevant diagrams are shown in fig. 1.

(a)

(b) (c)

Figure 1: Diagrams with virtual and real fermions contributing to top's width.

Calculation of the fermionic contributions is particularly important because it helps to establish the correct mass scale at which the strong coupling constant should be taken for the calculation of one-loop corrections.

We note that the relevant expansion parameter for the calculations of the decay width $\Gamma(t \to bW)$ is $m_W^2/m_t^2 \lesssim 0.2$. Therefore taking $m_W = 0$, as well as neglecting masses of all fermions except the decaying top quark as in ref. 2 is a very good approximation. The purpose of this paper is to show a method in which the fermionic subset of two-loop corrections can be calculated analytically.

2 Real and virtual contributions

The tree level decay rate of a top quark into massless W and b is (we take the relevant Kobayashi-Maskawa matrix element equal 1)

$$\Gamma^0 = \frac{G_F m_t^3}{\sqrt{2}} (2\pi)^{2-D} R_2 \tag{1}$$

where m_t is the pole mass of the top quark (we shall use it as the unit of mass throughout this calculation) and R_2 denotes the volume of the two-body phase space

$$R_2 \equiv \frac{\pi^{1-\varepsilon}\Gamma(1-\varepsilon)}{2\Gamma(2-2\varepsilon)} \tag{2}$$

In order to regularize infrared and ultraviolet divergences we perform the calculations in $D=4-2\varepsilon$ dimensions. In particular we need to know the one-loop gluonic corrections to $\Gamma(t\to bW)$ including terms $\mathcal{O}(\varepsilon)$. We find (neglecting terms containing the Euler constant γ_E and $\ln 4\pi$ which vanish in the final result)

$$\Gamma^1 = -\frac{G_F R_2 g_s^2 C_F}{\sqrt{2}2^{2D-3}\pi^{\frac{3}{2}D-2}}\left[-\frac{5}{2}+\frac{2}{3}\pi^2+\left(-\frac{61}{4}+\frac{5}{3}\pi^2+8\zeta(3)\right)\varepsilon\right]+\mathcal{O}(\varepsilon^2) \quad (3)$$

where $C_F=4/3$ is the $SU(3)$ color factor. We can now proceed to the calculation of contributions of a single flavor of light quarks to $\Gamma(t\to bW)$. We first consider the effect of the virtual correction to the vertex, as shown in fig. 1a. The contribution of a massless quark loop to the gluon propagator is (see e.g. [3])

$$\begin{aligned}\Pi^{\mu\nu} &= \Pi(q^2)\left(q^2 g^{\mu\nu}-q^\mu q^\nu\right)\\ \Pi(q^2) &= -\frac{ig_s^2}{(4\pi)^{D/2}}\frac{2-2\varepsilon}{3-2\varepsilon}\Gamma(\varepsilon)B(1-\varepsilon,1-\varepsilon)(-q^2)^{-\varepsilon}\end{aligned} \quad (4)$$

The vertex diagram can now be calculated exactly in any dimension

$$\begin{aligned}\Gamma^V &= -\frac{G_F C_F g_s^4 R_2}{\sqrt{2}2^{3D-4}\pi^{2D-2}}\frac{(1-\varepsilon)(2-9\varepsilon+23\varepsilon^2-30\varepsilon^3+16\varepsilon^4)}{3(1-2\varepsilon)^2(1-3\varepsilon)(2-3\varepsilon)(3-2\varepsilon)}\\ &\quad\times\frac{\Gamma^2(-\varepsilon)\Gamma(\varepsilon)\Gamma(2\varepsilon)\Gamma(-4\varepsilon)}{\Gamma(-2\varepsilon)\Gamma(-3\varepsilon)}\\ &\approx \frac{G_F R_2}{\sqrt{2}}\frac{g_s^4 C_F}{2^{3D-4}\pi^{2D-2}}\\ &\quad\times\left[\frac{1}{12\varepsilon^3}+\frac{11}{36\varepsilon^2}+\left(\frac{157}{108}+\frac{5\pi^2}{72}\right)\frac{1}{\varepsilon}+\frac{3733}{648}+\frac{55}{216}\pi^2+\frac{11}{18}\zeta(3)\right]\end{aligned} \quad (5)$$

Similarly, the correction to the external quark leg gives

$$\begin{aligned}\Gamma^Z &= -\frac{G_F R_2}{\sqrt{2}}\frac{g_s^4 C_F}{2^{3D-5}\pi^{2D-2}}\frac{\varepsilon(1-\varepsilon^2)}{3(1-3\varepsilon)(2-3\varepsilon)}\frac{\Gamma(\varepsilon)\Gamma^2(-\varepsilon)\Gamma(2\varepsilon)\Gamma(-4\varepsilon)}{\Gamma(-2\varepsilon)\Gamma(-3\varepsilon)}\\ &\approx \frac{G_F R_2}{\sqrt{2}}\frac{g_s^4 C_F}{2^{3D-4}\pi^{2D-2}}\left(\frac{1}{4\varepsilon^2}+\frac{9}{8\varepsilon}+\frac{59}{16}+\frac{5\pi^2}{24}\right)\end{aligned} \quad (6)$$

It is more difficult to calculate the effect of real quarks in the final state shown in fig. 1b,c. Especially the square of the amplitude of the gluon emission off the decaying quark and the interference between amplitudes with emissions from both quarks are troublesome. This is because the integration over the four particle phase space in the presence of the massive propagator in the diagrams leads in D dimensions to hypergeometric functions. However, for our aims it is sufficient to expand the result in a Laurent series in ε. We find

$$\begin{aligned}\Gamma^R &= -\frac{G_F R_2}{\sqrt{2}}\frac{g_s^4 C_F}{2^{3D-4}\pi^{2D-2}}\\ &\quad\times\left[\frac{1}{12\varepsilon^3}+\frac{5}{9\varepsilon^2}+\left(\frac{737}{216}-\frac{11}{72}\pi^2\right)\frac{1}{\varepsilon}+\frac{19985}{1296}-\frac{14}{27}\pi^2-\frac{73}{18}\zeta(3)\right]+\mathcal{O}(\varepsilon)\end{aligned} \quad (7)$$

3 Results and summary

It is interesting to look closer at the cancellation of divergences among the diagrams calculated in the previous section. The most singular, $1/\varepsilon^3$ terms, which have a purely infrared origin, vanish after summing the vertex and the real radiation diagrams (in fact only the interference of the two real radiation amplitudes is relevant at this level). The sum of the two formulas $\Gamma^V + \Gamma^R$ contains $1/\varepsilon^2$ poles which are cancelled after adding the external leg correction Γ^Z. The singularity of the resulting formula is proportional to the finite part of the one-loop gluonic correction Γ^1 and is removed by expressing the one-loop result in terms of the unrenormalized coupling constant. Of course at this stage we have the freedom of choosing the finite normalization of α_s. Various choices have been discussed recently [2, 4]. In order to better understand the terminology involved we look again at the gluon vacuum polarization in eq. (4). Expanding the numerical factor in this formula in ε we get

$$\Pi(q^2) = -\frac{4ig_s^2}{(4\pi)^{D/2}}\left(\frac{1}{6\varepsilon} + \frac{5}{18} + \mathcal{O}(\varepsilon)\right)(-q^2)^{-\varepsilon} \tag{8}$$

The $\overline{\mathrm{MS}}$ scheme corresponds to taking only the divergent part of this expansion for the renormalization of the coupling constant. In this scheme our result reads

$$\Gamma^{\mathrm{ferm}}(t \to Wb) = \left(\frac{\alpha_s}{\pi}\right)^2 \frac{N_f}{3}\Gamma^0\left[-\frac{8}{9} + \frac{23}{9}\zeta(2) + 2\zeta(3)\right] \tag{9}$$

where $\zeta(2) = \pi^2/6$ and $\zeta(3) \approx 1.2020569$, and N_f is the number of light flavors of quarks. The authors of ref. 2 recommend using the so-called V scheme [5] which in our case amounts to taking $-q^2 = 1$ in eq. 8 (we use m_t as the unit of mass). Our final result then becomes

$$\Gamma^{\mathrm{ferm}}(t \to Wb) = \left(\frac{\alpha_s}{\pi}\right)^2 \frac{N_f}{3}\Gamma^0\left[\frac{1}{2} + \frac{\zeta(2)}{3} + 2\zeta(3)\right] \tag{10}$$

Evaluation of the square bracket gives 3.452425... which confirms the numerical result given in ref. 2

$$2.54\left(\frac{2\pi^2}{9} - \frac{5}{6}\right) = 3.45\ldots \tag{11}$$

The analytical result given in eq. (10) describes the two-loop $\mathcal{O}(N_f\alpha_s^2)$ correction to the decay of the top quark into massless b quark and W boson. It is possible to extend this calculation to include the effects of both masses. In order to avoid difficulties connected with integrals over a four particle massive phase space I would recommend employing the method of asymptotic expansions (see e.g. [6]). This would reduce the problem to calculating derivatives of the amplitudes with respect to m_b and m_W in the massless limit. Therefore, one would obtain the same integral structures with possible higher powers of propagators.

Acknowledgements

I gratefully acknowledge helpful discussions with M. Buza, M. Jeżabek, and Q.P. Xu. This research was supported by a grant BMFT 056KA93P.

References

[1] M. Jeżabek and J.H. Kühn. *Nucl. Phys.* **B314** (1989) 1.

[2] B.H. Smith and M.B. Voloshin. *Phys. Lett.* **B340** (1994) 176.

[3] T. Muta. *Foundations of Quantum Chromodynamics.* World Scientific, Singapore, 1987.

[4] M. Shifman and N.G. Uraltsev. A closer look at perturbative corrections in the $b \to c$ semileptonic transitions, hep-ph/9412398.

[5] S.J. Brodsky, G.P. Lepage, and P.B. Mackenzie. *Phys. Rev.* **D28** (1983) 228.

[6] V.A. Smirnov. *Renormalization and asymptotic expansions.* Birkhäuser, 1991.